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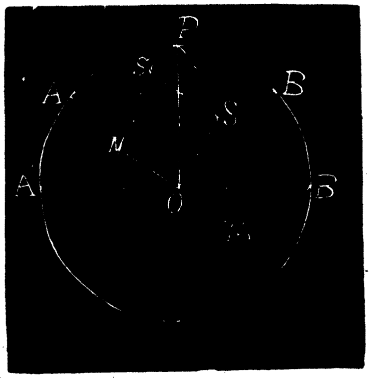
what latitude would the maximum distance be saved by a ship sailing on the arc of a great circle instead of a parallel of latitude, the points of departure and destination being 180° apart? Also find the maximum saving.

I. **Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and E. D. SCALES, Student of Junior Class, University of Mississippi, University, Mississippi.**

Put $\angle MOS=\lambda$, = the latitude of the *departure-point* of the ship. According to the problem, the ship departing from S may reach the destination S' by taking either the *longer* route $SAS'=(\pi\cos\lambda)R$, or by taking the *shorter* route $SPS'=(180^\circ-2\lambda)R$. The expression for the number of miles saved by taking the route SPS', becomes $M=2(\lambda-\pi\sin^2\tfrac{1}{2}\lambda)R$....(1), which is to be a maximum. Differentiating, etc., $\lambda=\sin^{-1}(2/\pi)$, $=39^\circ\ 32'\ 24''.784$; and, consequently, the number of miles saved is $M=[2\sin^{-1}(2/\pi)-\pi+\sqrt{(\pi^2-4)}]R$, $=2620.80359+$ English miles.



II. **Solution by O. W. ANTHONY, M. S., Missouri Military Academy, Mexico, Missouri, G. B. M. ZERR, A. M., Staunton, Virginia, and the PROPOSER.**

Let λ be the latitude and regard the earth as a perfect sphere with the same volume.

Then the radius of the small circle in latitude λ is $R\cos\lambda$, where R = radius of the earth.

$\therefore\ \pi R\cos\lambda$ is the distance to sail on a parallel of latitude and $(\pi-2\lambda)R$, the distance on a great circle.

$\therefore\ \pi R\cos\lambda-(\pi-2\lambda)R=\text{max.}$

$\therefore\ \pi\cos\lambda-\pi+2\lambda=\text{max.}$

Integrating, we get $\lambda=\sin^{-1}\dfrac{2}{\pi}=39^\circ\ 32'\ 24''.55$. The saving $=R(\pi\cos\lambda-\pi+2\lambda)$. But $R=20902410$ feet according to Col. Clark.

$\therefore$ the saving $=20902410\times.671385=14033564.53785$ feet

$=2657.87207$ miles.

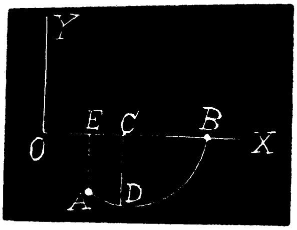
Also solved by *A. H. BELL, Hillsboro, Illinois*, and *E. W. MORRELL, Montpelier Seminary, Montpelier, Vermont.*

33. **Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, California.**

Show that of all curves of a given length, traced on one plane between two given points, and are made to revolve around a common axis situated in that plane, the Catenary generates a minimum area.

I. **Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Let A, B be the two given points, arc $ADB=S$, OBX the common axis, $OB=a$, $OE=b$, $OC=x$, $CD=y$. Then from Calculus of Variations, we get:—



$$\int Vdx=\int_b^a 2\pi y\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx=\text{a minimum}$$

and $\int V'dx=\int_b^a\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx=S.$

$$\therefore DU=D\int(V+\beta V')dx=0.$$

$$V+\beta V'=(2\pi y+\beta)\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}},=\text{also } P_1\frac{dy}{dx}+c$$

$$\text{where } P_1=(2\pi y+\beta)\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}\frac{dy}{dx}.$$

$$\therefore C+(2\pi y+\beta)\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}\left(\frac{dy}{dx}\right)^2=(2\pi y+\beta)\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}.$$

$$\therefore dx=\frac{cdy}{\sqrt{(2\pi y+\beta)^2-c^2}}.\quad \text{Let } 2\pi y+\beta=z,\text{ then } dy=\frac{dz}{2\pi}.$$

$$\therefore dx=\frac{c}{2\pi}\frac{dz}{\sqrt{z^2-c^2}},\ x=\frac{c}{2\pi}\log\frac{C}{z-\sqrt{z^2-c^2}}=\frac{c}{2\pi}\log\frac{C(z+\sqrt{z^2-c^2})}{c^2}.$$

$$\therefore x=\frac{c}{2\pi}\log\frac{C\{2\pi y+\beta+\sqrt{(2\pi y+\beta)^2-c^2}\}}{c^2}.$$

$$\therefore y=\frac{1}{4\pi C}\left(c^2e^{\frac{2\pi x}{c}}+C^2e^{-\frac{2\pi x}{c}}\right)-\frac{\beta}{2\pi},\text{ the equation of a Catenary.}$$

II. Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering, A. and M. College, College Station, Texas.

It is shown in Statics that of all curves of given length between two given points the Catenary has its center of gravity lower than any other—the curve being converse to the axis, and gravity acting at right angles thereto (See Minchin's Statics, Vol. II, page 161).

Now by the theorem of Pappus the area generated equals the length of curve into path described by the center of gravity in turning about the axis; and since the radius of the circle described by the *c. g.* of the Catenary is less than for any other curve considered it is evident the area generated is a minimum.

If the Catenary is concave to the axis of x the area generated will be a maximum, and is so proven in Duhamel's "Elements de Calcul Infinitesimal, Tome Second", page 400.

Prof. *MATZ*, sent three excellent solutions to the above problem.